

Finite Time and Aperiodically Time Dependent Dynamics: The Research Landscape— Past, Present, and Future

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Over the past 30 years the dynamical systems approach to transport in fluids has experienced continual growth, both in the scope of applications and in the development of computational and mathematical methods. Realistic velocity fields are not periodic in time (i.e. they are “aperiodic”) and are often defined as “data sets”, which results in them only being defined for a finite time. Both of these features-- aperiodic time dependence and being only defined on a finite time interval--pose new computational and mathematical challenges to the dynamical systems approach to transport, and the mathematical development of dynamical systems theory in general.

In my talk I will discuss these issues in an informal setting. To begin with, I will give an overview of some of the main issues, and describe relevant work that does not appear known (and discuss some reasons for that). I will pay particular attention here to the “hyperbolic-elliptic” dichotomy in Hamiltonian systems.

Kolmogorov-Arnold-Moser (KAM) tori are elliptic invariant manifolds that act as “complete” barriers to transport. While generalizing theorems on “hyperbolic objects and phenomena” is relatively straightforward (indeed, this was done many years before the dynamical systems approach to transport was developed, as I will describe) generalizing theorems related to “elliptic objects and phenomena”, such as KAM tori, has proven much more difficult. Nekhoroshev’s theorem is a finite time stability result that can be used to give meaning to the notion of an “almost invariant” torus. The finite time interval on which the estimates hold is exponentially long with respect to certain parameters of the system. In the words of Littlewood, “..while not eternity, this is a considerable slice of it”. I will explain the idea behind these exponential estimates, and I will also discuss the relation between the KAM and Nekhoroshev theorems and describe these results for general, aperiodic time dependence.

I will discuss an example of a two-dimensional, aperiodically time-dependent velocity fields where it can be shown analytically that it possesses invariant tori. This allows one to see explicitly how invariant tori in aperiodically time-dependent velocity fields are manifested, and is an ideal “laboratory” for future studies of all of the issues that I have raised concerning hyperbolic versus elliptic phenomena and finite time versus infinite time dynamics.

Finally, topics of my talk raise a number of issues about how research areas develop, and they also point to a number of very interesting research problems that have been overlooked and would be of great interest in a number of applications. I will discuss several of these explicitly.